WINTER – 2015 EXAMINATION

MODEL ANSWER

Subject: ENGINEERING MATHEMATICS (EMS)

Subject Code: 17216

Important Instructions to examiners:

- The model answer shall be the complete solution for each and every question on the question paper.
- Numerical shall be completely solved in a step by step manner along with step marking.
- All alternative solutions shall be offered by the expert along with self-explanatory comments from the expert.
- In case of theoretical answers, the expert has to write the most acceptable answer and offer comments regarding marking scheme to the assessors.
- In should offer the most convincing figures / sketches / circuit diagrams / block diagrams / flow diagrams and offer comments for step marking to the assessors.
- In case of any missing data, the expert shall offer possible assumptions / options and the ensuing solutions along with comments to the assessors for effective assessment.
- In case of questions which are out of the scope of curricular requirement, the expert examiner shall solve the question and mention the marking scheme in the model answer. However, the experts are requested to submit their clear cut opinion about the scope of such question in the paper separately to the coordinator.
- Experts shall cross check the DTP of the final draft of the model answer prepared by them.

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)		**Attempt any <u>TEN</u> of the following:**		20
	(a)	If $z = 1 + 3i$, evaluate $z^2 + 2z + 4$		
	Ans.	$z^2 + 2z + 4$		
		$= (1+3i)^2 + 2(1+3i) + 4$	½	
		$= 1 + 6i - 9 + 2 + 6i + 4$	1	02
		$= -2 + 12i$	½	
		--		
	(b)	Express $1+i$ in modulus and amplitude form		
	Ans.	Let $z = 1 + i$		
		$\therefore\ x = 1, y = 1$		
		$r = \lvert z \rvert = \sqrt{x^2 + y^2} = \sqrt{1+1} = \sqrt{2}$	½	
		and $x, y > 0$	1	
		$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \tan^{-1}\left(\frac{1}{1}\right) = \tan^{-1}(1) = \frac{\pi}{4}$		
		$\therefore\ z = r(\cos\theta + i\sin\theta)$	½	02
		$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$		
		--		
	c)	If $f(x) = 16^x + \log_4 x$, find $f\left(\frac{1}{2}\right)$		
	Ans.	$f(x) = 16^x + \log_4 x$,		
		$\therefore\ f\left(\frac{1}{2}\right) = (16)^{\frac{1}{2}} + \log_4\left(\frac{1}{2}\right)$	½	
		$= 4 - \log_4 2$		
		$= 4 - \frac{\log 2}{\log 4}$	½	
		$= 4 - \frac{\log 2}{2\log 2}$		
		$= 4 - \frac{1}{2}$	½	02
		$= \frac{7}{2}$	½	
		--		

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
1.	d)	Define even and odd function		
	Ans.	Even function:- If $f(-x) = f(x)$, then the function is an even function	1	
		Odd function:- If $f(-x) = -f(x)$, then the function is an odd function	1	02
	(e)	Evaluate: $\lim_{x \to 1} \frac{x^2 + 2x + 5}{x + 1}$		
	Ans	$\lim_{x \to 1} \frac{x^2 + 2x + 5}{x + 1}$		
		$= \frac{(1)^2 + 2(1) + 5}{1 + 1}$	1	
		$= \frac{8}{2}$	½	02
		$= 4$	½	
	(f)	Evaluate: $\lim_{x \to 0} \frac{\sin 3x}{\tan 5x}$		
	Ans.	$\lim_{x \to 0} \frac{\sin 3x}{\tan 5x}$		
		$= \lim_{x \to 0} \frac{\frac{\sin 3x}{x}}{\frac{\tan 5x}{x}}$	½	
		$= \frac{\left(\lim_{x \to 0} \frac{\sin 3x}{3x}.3\right)}{\left(\lim_{x \to 0} \frac{\tan 5x}{5x}.5\right)}$	½	
		$= \frac{(1)3}{(1)5}$	½	02
		$= \frac{3}{5}$	½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1.	g)	Evaluate: $\lim_{x\to 0}\frac{3^{2x}-2^{3x}}{\sin x}$		
	Ans	$\lim_{x\to 0}\frac{3^{2x}-2^{3x}}{\sin x}$		
		$=\lim_{x\to 0}\frac{\frac{\left(3^{2x}-1-2^{3x}+1\right)}{x}}{\frac{\sin x}{x}}$	½	
		$=\frac{\lim_{x\to 0}\left(\frac{\left(3^{2x}-1\right)-\left(2^{3x}-1\right)}{x}\right)}{\lim_{x\to 0}\frac{\sin x}{x}}$	½	
		$=\frac{\lim_{x\to 0}\left(\frac{\left(3^{2x}-1\right)}{x}-\frac{\left(2^{3x}-1\right)}{x}\right)}{\lim_{x\to 0}\frac{\sin x}{x}}$		
		$=\frac{\left(\lim_{x\to 0}\frac{3^{2x}-1}{2x}\right).2-\left(\lim_{x\to 0}\frac{2^{3x}-1}{3x}\right).3}{\lim_{x\to 0}\frac{\sin x}{x}}$	½	
		$=\frac{2\log 3-3\log 2}{1}$	½	
		$=\log 9-\log 8=\log\left(\frac{9}{8}\right)$		02
		--		
	(h)	If $y=e^{4x}\cos 3x$, find $\frac{dy}{dx}$		
	Ans	$y=e^{4x}\cos 3x$		
		$\frac{dy}{dx}=e^{4x}\left(-\sin 3x\right)3+\cos 3x\ e^{4x}.4$	1+1	
		$\frac{dy}{dx}=e^{4x}\left(-3\sin 3x+4\cos 3x\right)$		02
		--		
	(i)	If $y=\log\left[\sin\left(4x-3\right)\right]$, find $\frac{dy}{dx}$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
1)	Ans	$y = \log\left[\sin(4x-3)\right]$		
		$\frac{dy}{dx} = \frac{1}{\sin(4x-3)}\cos(4x-3)4$ OR $\frac{dy}{dx} = \frac{1}{\sin(4x-3)}\frac{d}{dx}\left[\sin(4x-3)\right]$	1+½	
		$\frac{dy}{dx} = \frac{4\cos(4x-3)}{\sin(4x-3)}$ OR $\frac{dy}{dx} = \frac{\cos(4x-3)}{\sin(4x-3)}\frac{d}{dx}(4x-3)$		
		$\frac{dy}{dx} = 4\cot(4x-3)$ OR $\frac{dy}{dx} = 4\cot(4x-3)$	½	02
		---		
	j)	Find $\frac{dy}{dx}$, if $x = 4\sin 3\theta,\ y = 4\cos 6\theta$		
	Ans	$x = 4\sin 3\theta,\ y = 4\cos 6\theta$		
		$\frac{dx}{d\theta} = 12\cos 3\theta$ and $\frac{dy}{d\theta} = -24\sin 6\theta$	½ +½	
		$\because \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{-24\sin 6\theta}{12\cos 3\theta}$	1	
		$\frac{dy}{dx} = \frac{-2.2\sin 3\theta\cos 3\theta}{\cos 3\theta}$		
		$\frac{dy}{dx} = -4\sin 3\theta$		02
		---		
	k)	Show that the root of $x^3 - 9x + 1 = 0$ lies between 2 and 3.		
	Ans	Let $f(x) = x^3 - 9x + 1$		
		$f(2) = -9 < 0$	1	
		$f(3) = 1 > 0$	1	02
		$\therefore$ root lies between 2 and 3		
	l)	Find the first iteration by using Jacobi's method for the following system of equations: $5x + 2y + z = 12,\ x + 4y + 2z = 15,\ x + 2y + 5z = 20$		
	Ans	Initial approximations : $x_0 = y_0 = z_0 = 0$		
		$x = \frac{12 - 2y - z}{5}$		
		$y = \frac{15 - x - 2z}{4}$		
		$z = \frac{20 - x - 2y}{5}$	½	
		$x = 2.4$, $y = 3.75$, $z = 4$	1 ½	02

2.		**Attempt any FOUR of the following:**		16
	a)	Find cube root of unity and show that one root is square of the other.		
	Ans	$w=\sqrt[3]{1}$ $\therefore\ w^3=1$ Put $w^3=z$ $\therefore\ z=1+0i$ $x=1>0,\ y=0$ $r=\lvert z\rvert=\sqrt{1+0}=1$ $\theta=\tan^{-1}\left(\frac{0}{1}\right)=0$ General polar form is, $z=r\left(\cos\left(2n\pi+\theta\right)+i\sin\left(2n\pi+\theta\right)\right)$ $w^3=1\left(\cos 2n\pi+i\sin 2n\pi\right)$ $w=\left(\cos 2n\pi+i\sin 2n\pi\right)^{\frac{1}{3}}$ $w=\cos\left(\frac{2n\pi}{3}\right)+i\sin\left(\frac{2n\pi}{3}\right)\quad;\quad n=0,1,2$ when $n=0$ $w_1=\cos 0+i\sin 0=1$ when $n=1$ $w_2=\cos\left(\frac{2\pi}{3}\right)+i\sin\left(\frac{2\pi}{3}\right)=-\frac{1}{2}+i\frac{\sqrt{3}}{2}$ when $n=2$ $w_3=\cos\left(\frac{4\pi}{3}\right)+i\sin\left(\frac{4\pi}{3}\right)=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$ consider $\left(w_2\right)^2=\left(-\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)^2$ $=\frac{1}{4}-i\frac{\sqrt{3}}{2}-\frac{3}{4}$ $=-\frac{1}{2}-i\frac{\sqrt{3}}{2}$ $=w_3$	½ ½ ½ ½ ½ ½ ½ ½	04
		--		
	b)	Simplify: $\frac{\left(\cos 2\theta+i\sin 2\theta\right)\left(\cos\theta-i\sin\theta\right)^4}{\left(\cos 3\theta+i\sin 3\theta\right)\left(\cos 5\theta-i\sin 5\theta\right)^3}$ using De-Moiver's theorem		

<table>
<tr><th>Que. No.</th><th>Sub. Que.</th><th>Model Answers</th><th>Marks</th><th>Total Marks</th></tr>
<tr><td>2)</td><td>Ans</td><td>$\frac{(\cos 2\theta + i\sin 2\theta)(\cos\theta - i\sin\theta)^4}{(\cos 3\theta + i\sin 3\theta)(\cos 5\theta - i\sin 5\theta)^3}$
$= \frac{(\cos\theta + i\sin\theta)^2(\cos\theta + i\sin\theta)^{-4}}{(\cos\theta + i\sin\theta)^3(\cos\theta + i\sin\theta)^{-15}}$
$= (\cos\theta + i\sin\theta)^{2-4-3+15}$
$= (\cos\theta + i\sin\theta)^{10}$
$= \cos 10\theta + i\sin 10\theta$
---</td><td>½
+½+½
+½
1
1</td><td>04</td></tr>
<tr><td></td><td>c)

Ans</td><td>If $\sin(A+iB) = x+iy$ prove that:
i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = 1$
ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = 1$
$\sin(A+iB) = x+iy$
$\sin A\cos(iB) + \cos A\sin(iB) = x+iy$
$\sin A\cosh B + i\cos A\sinh B = x+iy$
$\therefore x = \sin A\cosh B, y = \cos A\sinh B$
i) $\frac{x^2}{\cosh^2 B} + \frac{y^2}{\sinh^2 B} = \frac{\sin^2 A\cosh^2 B}{\cosh^2 B} + \frac{\cos^2 A\sinh^2 B}{\sinh^2 B}$
$= \sin^2 A + \cos^2 A$
$= 1$
ii) $\frac{x^2}{\sin^2 A} - \frac{y^2}{\cos^2 A} = \frac{\sin^2 A\cosh^2 B}{\sin^2 A} - \frac{\cos^2 A\sinh^2 B}{\cos^2 A}$
$= \cosh^2 B - \sinh^2 B$
$= 1$
---</td><td>1
1
½
1
½</td><td>04</td></tr>
<tr><td></td><td>d)
Ans</td><td>If $f(x) = \log\left(\frac{x}{x-1}\right)$ show that $f(a+1) + f(a) = \log\left(\frac{a+1}{a-1}\right)$
$f(a+1) + f(a) = \log\left(\frac{a+1}{a+1-1}\right) + \log\left(\frac{a}{a-1}\right)$
$= \log\left(\frac{a+1}{a}\cdot\frac{a}{a-1}\right)$
$= \log\left(\frac{a+1}{a-1}\right)$</td><td>1+1
1
1</td><td>04</td></tr>
</table>

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
2)	e)	Using Euler's exponential formula prove that: $\sin^2\theta+\cos^2\theta=1$		
	Ans	$\sin^2\theta+\cos^2\theta$		
		$=\left(\dfrac{e^{i\theta}-e^{-i\theta}}{2i}\right)^2+\left(\dfrac{e^{i\theta}+e^{-i\theta}}{2}\right)^2$		
		$=\dfrac{1}{4i^2}\left(e^{i\theta}-e^{-i\theta}\right)^2+\dfrac{1}{4}\left(e^{i\theta}+e^{-i\theta}\right)^2$	1	
		$=\dfrac{-1}{4}\left(e^{2i\theta}-2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)+\dfrac{1}{4}\left(e^{2i\theta}+2e^{i\theta}e^{-i\theta}+e^{-2i\theta}\right)$	1	
		$=\dfrac{1}{4}\left(4e^{i\theta}e^{-i\theta}\right)=\dfrac{1}{4}\left(4e^{0}\right)$	1	
		$=1$	1	04
		---		
	f)	If $f(x)=\dfrac{3x+2}{4x-3}$ show that $f=f^{-1}$		
	Ans	Let $f(x)=\dfrac{3x+2}{4x-3}$		
		consider		
		$fof(x)=f\left[f(x)\right]$	1	
		$=f\left[\dfrac{3x+2}{4x-3}\right]$	½	
		$=\dfrac{3\left(\dfrac{3x+2}{4x-3}\right)+2}{4\left(\dfrac{3x+2}{4x-3}\right)-3}$	½	
		$=\dfrac{3(3x+2)+2(4x-3)}{4(3x+2)-3(4x-3)}$	1	
		$=\dfrac{17x}{17}$		
		$fof(x)=x$		
		$\therefore\ f(x)=f^{-1}(x)$		04
		$f=f^{-1}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		**Attempt any <u>FOUR</u> of the following:**		16
	a)	If $f(x)=\dfrac{x+3}{4x-5}$ and $t=\dfrac{3+5x}{4x-1}$ show that $f(t)=x$		
	Ans	$f(t)=\dfrac{t+3}{4t-5}$	½	
		$=\dfrac{\dfrac{3+5x}{4x-1}+3}{4\left(\dfrac{3+5x}{4x-1}\right)-5}$	1	
		$=\dfrac{\dfrac{3+5x+3(4x-1)}{4x-1}}{\dfrac{4(3+5x)-5(4x-1)}{4x-1}}$	1	
		$=\dfrac{3+5x+12x-3}{12+20x-20x+5}$	½	
		$=\dfrac{17x}{17}=x$	1	04
		---		
	b)	If $f(t)=50\sin(100\pi t+0.04)$, then show that $f\left(\dfrac{2}{100}+t\right)=f(t)$		
	Ans	$f\left(\dfrac{2}{100}+t\right)=50\sin\left(100\pi\left(\dfrac{2}{100}+t\right)+0.04\right)$	1	
		$=50\sin(2\pi+100\pi t+0.04)$	1	
		$=50\sin(100\pi t+0.04)$	1	
		$=f(t)$	1	04
		---		
	c)	Evaluate: $\lim\limits_{x\to 0}\dfrac{\sqrt{3+x}-\sqrt{3}}{x}$		
	Ans	$=\lim\limits_{x\to 0}\dfrac{\sqrt{3+x}-\sqrt{3}}{x}\cdot\dfrac{\sqrt{3+x}+\sqrt{3}}{\sqrt{3+x}+\sqrt{3}}$	1	
		$=\lim\limits_{x\to 0}\dfrac{3+x-3}{x\left(\sqrt{3+x}+\sqrt{3}\right)}$	1	
		$=\dfrac{1}{\sqrt{3+0}+\sqrt{3}}$	1	
		$=\dfrac{1}{2\sqrt{3}}$	1	04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)	d)	Evaluate: $\lim\limits_{x\to 0}\dfrac{\sin 2x-2\sin x}{x^3}$		
	Ans	$=\lim\limits_{x\to 0}\dfrac{2\sin x\cos x-2\sin x}{x^3}$	½	
		$=\lim\limits_{x\to 0}\dfrac{-2\sin x\left(1-\cos x\right)}{x^3}$	½	
		$=\lim\limits_{x\to 0}\dfrac{-2\sin x\,2\sin^2\left(\dfrac{x}{2}\right)}{x^3}$	½	
		$=-4\lim\limits_{x\to 0}\dfrac{\sin x}{x}\dfrac{\sin^2\left(\dfrac{x}{2}\right)}{x^2}$		
		$=-4\left(\lim\limits_{x\to 0}\dfrac{\sin x}{x}\right)\left(\lim\limits_{x\to 0}\dfrac{\sin\left(\dfrac{x}{2}\right)}{\dfrac{x}{2}}.\dfrac{1}{2}\right)^2$	1	
		$=-4\left(1\right)\left(1.\dfrac{1}{2}\right)^2$	1	04
		$=-\dfrac{4}{4}=-1$	½	
		---		
	e)	Evaluate: $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\cos 3x+3\cos x}{\left(\dfrac{\pi}{2}-x\right)^3}$		
	Ans	$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{4\cos^3 x-3\cos x+3\cos x}{\left(\dfrac{\pi}{2}-x\right)^3}$	½	
		$=\lim\limits_{x\to\frac{\pi}{2}}\dfrac{4\cos^3 x}{\left(\dfrac{\pi}{2}-x\right)^3}$		
		Put $x=\dfrac{\pi}{2}+h$, as $x\to\dfrac{\pi}{2}$, $h\to 0$	1	
		$=\lim\limits_{h\to 0}\dfrac{4\cos^3\left(\dfrac{\pi}{2}+h\right)}{\left(-h\right)^3}$	1	
		$=-4\lim\limits_{h\to 0}\dfrac{\sin^3 h}{-h^3}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
3)		$= 4\left(\lim_{h\to 0}\frac{\sin h}{h}\right)^3$		
		$= 4(1)^3$		
		$= 4$	½	04
		--		
	f)	Evaluate : $\lim_{x\to 3}\frac{\log(x-2)}{x^2-9}$		
	Ans	Put $x = 3+h$, as $x\to 3, h\to 0$	1	
		$= \lim_{h\to 0}\frac{\log(3+h-2)}{(3+h)^2-9}$		
		$= \lim_{h\to 0}\frac{\log(1+h)}{9+6h+h^2-9}$	1	
		$= \lim_{h\to 0}\frac{\log(1+h)}{h(6+h)}$		
		$= \lim_{h\to 0}\frac{\log(1+h)^{\frac{1}{h}}}{(6+h)}$	1	
		$= \frac{\log e}{6}$	½	
		$= \frac{1}{6}$	½	04
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**Attempt any <u>FOUR</u> of the following:**		16
	a)	If u and v are differentiable functions of x and $y=\dfrac{u}{v}$, where $v\neq 0$ then prove that $\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}$		
	Ans	Let $\delta u,\delta v,\delta y$ are small increments in u,v,y respectively corresponding to increment δx in x.		
		$\therefore\ y+\delta y=\dfrac{u+\delta u}{v+\delta v}$	½	
		$\delta y=\dfrac{u+\delta u}{v+\delta v}-y$		
		$\delta y=\dfrac{u+\delta u}{v+\delta v}-\dfrac{u}{v}$	½	
		$\delta y=\dfrac{uv+v\delta u-u(v+\delta v)}{v(v+\delta v)}$	½	
		$\delta y=\dfrac{v\delta u-u\delta v}{v^2+v\delta v}$		
		$\dfrac{\delta y}{\delta x}=\dfrac{\dfrac{v\delta u-u\delta v}{\delta x}}{v^2+v\delta v}$		
		$\lim\limits_{\delta x\to 0}\dfrac{\delta y}{\delta x}=\lim\limits_{\delta x\to 0}\dfrac{\dfrac{v\delta u-u\delta v}{\delta x}}{v^2+v\delta v}$	1	
		$\lim\limits_{\delta x\to 0}\dfrac{\delta y}{\delta x}=\dfrac{v\lim\limits_{\delta x\to 0}\dfrac{\delta u}{\delta x}-u\lim\limits_{\delta x\to 0}\dfrac{\delta v}{\delta x}}{v^2}$	½	
		$\dfrac{dy}{dx}=\dfrac{v\dfrac{du}{dx}-u\dfrac{dv}{dx}}{v^2}\qquad(\because \text{ as } \delta x\to 0,\delta v\to 0)$	1	04
		---		
	b)	By using first principle find the derivative of $y=\cos x$		
	Ans	$\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{f(x+h)-f(x)}{h}$		
		$\dfrac{dy}{dx}=\lim\limits_{h\to 0}\dfrac{\cos(x+h)-\cos x}{h}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		$\frac{dy}{dx}=\lim_{h\to 0}\frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h}$	1	
		$\frac{dy}{dx}=-2\lim_{h\to 0}\frac{\sin\left(\frac{2x+h}{2}\right)\sin\left(\frac{h}{2}\right)}{h}$		
		$\frac{dy}{dx}=-2\left(\lim_{h\to 0}\sin\left(\frac{2x+h}{2}\right)\right)\left(\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}.\frac{1}{2}\right)$	1	
		$\frac{dy}{dx}=-2\left(\sin x\right)\frac{1}{2}$	½	
		$\frac{dy}{dx}=-\sin x$	½	04
		---		
	c) Ans	If $y=\sin^{-1}\left[\frac{1}{\sqrt{1+x^2}}\right]$, find $\frac{dy}{dx}$		
		Put $x=\tan\theta \Rightarrow \tan^{-1}x=\theta$	½	
		$y=\sin^{-1}\left[\frac{1}{\sqrt{1+\tan^2\theta}}\right]$	½	
		$y=\sin^{-1}\left[\frac{1}{\sec\theta}\right]$		
		$y=\sin^{-1}\left[\cos\theta\right]$	1	
		$y=\sin^{-1}\left[\sin\left(\frac{\pi}{2}-\theta\right)\right]$		
		$y=\frac{\pi}{2}-\theta$	½	
		$y=\frac{\pi}{2}-\tan^{-1}x$	½	
		$\frac{dy}{dx}=0-\frac{1}{1+x^2}$	1	
		$\frac{dy}{dx}=-\frac{1}{1+x^2}$		04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)		**OR**		
		If $y=\sin^{-1}\left[\dfrac{1}{\sqrt{1+x^2}}\right]$, find $\dfrac{dy}{dx}$		
		Put $x=\cot\theta \Rightarrow \cot^{-1}x=\theta$	½	
		$y=\sin^{-1}\left[\dfrac{1}{\sqrt{1+\cot^2\theta}}\right]$	½	
		$y=\sin^{-1}\left[\dfrac{1}{\cos ec\theta}\right]$		
		$y=\sin^{-1}[\sin\theta]$	1	
		$y=\theta$	½	
		$y=\cot^{-1}x$	½	
		$\dfrac{dy}{dx}=-\dfrac{1}{1+x^2}$	1	04
		--		
	d) Ans	Find $\dfrac{dy}{dx}$ if $y=\dfrac{(\cos x)^x}{1+x^2}$		
		Given, $y=\dfrac{(\cos x)^x}{1+x^2}$		
		$\log y=\log\left[\dfrac{(\cos x)^x}{1+x^2}\right]$	½	
		$\log y=\log(\cos x)^x-\log(1+x^2)$	½	
		$\log y=x\log(\cos x)-\log(1+x^2)$	½	
		$\therefore \dfrac{1}{y}\dfrac{dy}{dx}=x\dfrac{1}{\cos x}(-\sin x)+\log(\cos x)-\dfrac{1}{1+x^2}2x$	2	
		$\dfrac{dy}{dx}=y\left(-x\tan x+\log(\cos x)-\dfrac{2x}{1+x^2}\right)$	½	
		$\dfrac{dy}{dx}=\dfrac{(\cos x)^x}{1+x^2}\left[-x\tan x+\log(\cos x)-\dfrac{2x}{1+x^2}\right]$		04
		--		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
4)	e) Ans	If $x^p.y^q=(x+y)^{p+q}$ show that $\frac{dy}{dx}=\frac{y}{x}$		
		$\log\left(x^p y^q\right)=\log(x+y)^{p+q}$	½	
		$\log x^p+\log y^q=(p+q)\log(x+y)$	½	
		$p\log x+q\log y=(p+q)\log(x+y)$		
		$p\frac{1}{x}+q\frac{1}{y}\frac{dy}{dx}=(p+q)\left[\frac{1}{x+y}\left(1+\frac{dy}{dx}\right)\right]$	1	
		$\frac{p}{x}+\frac{q}{y}\frac{dy}{dx}=\frac{p+q}{x+y}+\left(\frac{p+q}{x+y}\right)\frac{dy}{dx}$		
		$\left(\frac{q}{y}-\frac{p+q}{x+y}\right)\frac{dy}{dx}=\frac{p+q}{x+y}-\frac{p}{x}$	½	
		$\left(\frac{qx+qy-py-qy}{y(x+y)}\right)\frac{dy}{dx}=\frac{px+qx-px-py}{x(x+y)}$	½	
		$\left(\frac{qx-py}{y}\right)\frac{dy}{dx}=\frac{qx-py}{x}$	½	
		$\therefore\frac{dy}{dx}=\frac{y}{x}$	½	04
	f) Ans	If $y=3\sin t-2\sin^3 t,\ x=3\cos t-2\cos^3 t$, find $\frac{dy}{dx}$ at $t=\frac{\pi}{4}$		
		$y=3\sin t-2\sin^3 t,\ x=3\cos t-2\cos^3 t$		
		$\therefore\frac{dy}{dt}=3\cos t-6\sin^2 t\cos t$	½	
		$\frac{dx}{dt}=-3\sin t+6\cos^2 t\sin t$	½	
		$\therefore\frac{dy}{dx}=\frac{\frac{dy}{dt}}{\frac{dx}{dt}}=\frac{3\cos t-6\sin^2 t\cos t}{-3\sin t+6\cos^2 t\sin t}$	½	
		$\frac{dy}{dx}=\frac{3\cos t\left(1-2\sin^2 t\right)}{3\sin t\left(2\cos^2 t-1\right)}$	½	
		$\frac{dy}{dx}=\frac{\cos t\cos 2t}{\sin t\cos 2t}=\cot t$	1	
		at $t=\frac{\pi}{4}$		
		$\frac{dy}{dx}=\cot\frac{\pi}{4}=1$	1	04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		**Attempt any <u>FOUR</u> of the following:**		16
	a) Ans	Evaluate: $\lim_{x\to 0}\dfrac{6^x-3^x-2^x+1}{x^2}$		
		$=\lim_{x\to 0}\dfrac{6^x-3^x-2^x+1}{x^2}$		
		$=\lim_{x\to 0}\dfrac{3^x2^x-3^x-2^x+1}{x^2}$	½	
		$=\lim_{x\to 0}\dfrac{3^x\left(2^x-1\right)-\left(2^x-1\right)}{x^2}$		
		$=\lim_{x\to 0}\dfrac{\left(3^x-1\right)\left(2^x-1\right)}{x^2}$	1	
		$=\left(\lim_{x\to 0}\dfrac{3^x-1}{x}\right)\left(\lim_{x\to 0}\dfrac{2^x-1}{x}\right)$	½	
		$=\left(\log 3\right)\left(\log 2\right)$	1+1	04
		--		
	b) Ans	Evaluate: $\lim_{x\to 3}\dfrac{\log x-\log 3}{x-3}$		
		Put $x=3+h$ as $x\to 3, h\to 0$	1	
		$=\lim_{h\to 0}\dfrac{\log\left(3+h\right)-\log 3}{3+h-3}$		
		$=\lim_{h\to 0}\dfrac{\log\left(\dfrac{3+h}{3}\right)}{h}$	1	
		$=\lim_{h\to 0}\dfrac{1}{h}\log\left(1+\dfrac{h}{3}\right)$		
		$=\lim_{h\to 0}\log\left(1+\dfrac{h}{3}\right)^{\frac{1}{h}}$	½	
		$=\log\left[\lim_{h\to 0}\left(1+\dfrac{h}{3}\right)^{\frac{3}{h}}\right]^{\frac{1}{3}}$	1	
		$=\log e^{\frac{1}{3}}$		
		$=\dfrac{1}{3}\log e=\dfrac{1}{3}$	½	04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	c)	Find the approximate roots of the equation $x^3 - x - 4 = 0$ by bisection method.		
	Ans	Let $f(x) = x^3 - x - 4$ $f(1) = -4 < 0$ $f(2) = 2 > 0$ $\therefore$ root lies in $(1,2)$	1	
		$x_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$ $f(1.5) = -2.125 < 0$ $\therefore$ the root lies in (1.5,2)	1	
		$x_2 = \frac{x_1+b}{2} = \frac{1.5+2}{2} = 1.75$ $f(x_2) = -0.39 < 0$ $\therefore$ the root lies in (1.75,2)	1	
		$x_3 = \frac{x_2+b}{2} = \frac{1.75+2}{2} = 1.875$	1	04
		OR		
		Let $f(x) = x^3 - x - 4$ $f(1) = -4 < 0$ $f(2) = 2 > 0$ $\therefore$ root lies in $(1,2)$	1	
		(table below)	1+1+1	04
	d)	Show that root of the equation $x^3 - 4x + 1 = 0$ in $(1,2)$ and find it by using Newton-Raphson method performing two iterations.		

a	b	$x = \frac{a+b}{2}$	$f(x)$
1	2	1.5	-2.125
1.5	2	1.75	-0.39
1.75	2	1.875	---

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)	Ans	Let, $f(x) = x^3 - 4x + 1$		
		$f(1) = -2 < 0$		
		$f(2) = 1 > 0$	1	
		$f'(x) = 3x^2 - 4$	½	
		$\therefore\ f'(2) = 8$		
		$\therefore$ Initial root $x_0 = 2$		
		$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 2 - \dfrac{1}{8} = 1.875$	1 ½	
		$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 1.875 - \dfrac{0.091}{6.546} = 1.861$	1	04
		OR		
		Let, $f(x) = x^3 - 4x + 1$		
		$f(1) = -2 < 0$		
		$f(2) = 1 > 0$	1	
		$f'(x) = 3x^2 - 4$	½	
		$\therefore\ f'(2) = 8$		
		$\therefore$ Initial root $x_0 = 2$		
		$x_{n+1} = \dfrac{xf'(x) - f(x)}{f'(x)}$		
		$x_{n+1} = \dfrac{x(3x^2 - 4) - (x^3 - 4x + 1)}{3x^2 - 4}$		
		$x_{n+1} = \dfrac{3x^3 - 4x - x^3 + 4x - 1}{3x^2 - 4}$		
		$x_{n+1} = \dfrac{2x^3 - 1}{3x^2 - 4}$	1	
		$n = 0, 1, 2$		
		$x_1 = 1.875$	1	
		$x_2 = 1.86$	½	04

Que. No.	Sub. Que.	Model answers	Marks	Total Marks
5)	e)	Solve the following equations, Using Gauss elimination method: $x+2y+3z=14$, $3x+y+2z=11$, $2x+3y+z=11$		
	Ans	$x+2y+3z=14$ $3x+y+2z=11$ $2x+3y+z=11$		
		$x+2y+3z=14$ $6x+2y+4z=22$ $\underline{-\quad -\quad -\quad} -$ $-5x-z=-8$ and $9x+3y+6z=33$ $2x+3y+z=11$ $\underline{-\quad -\quad -} -$ $7x+5z=22$	1	
		$-25x-5z=-40$ $\underline{7x+5z=22}$ $-18x=-18$		
		$\therefore\ x=1$	1	
		$y=2$	1	04
		$z=3$	1	
		Note: *In the above solution, first y is eliminated and then z is eliminated to find the value of x first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking.*		
		---		
	f)	Solve the following equations by Gauss-Seidel method: $5x-y=9, x-5y+z=-4, y-5z=6$		
	Ans	$x=\frac{1}{5}(9+y)$ $y=\frac{1}{-5}(-4-x-z)$ $z=\frac{1}{-5}(6-y)$	1	
		Starting with $y_0=z_0=0$		
		$x_1=1.8$ $y_1=1.16$ $z_1=-0.968$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
5)		$x_2 = 2.032$ $y_2 = 1.012$ $z_2 = -0.997$	1	
		$x_3 = 2.003$ $y_3 = 1.001$ $z_3 = -0.9998$	1	04
6)		**Attempt any FOUR of the following:**		16
	a)	If $y = e^{m\sin^{-1}x}$ prove that $\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$		
	Ans	$y = e^{m\sin^{-1}x}$		
		$\frac{dy}{dx} = e^{m\sin^{-1}x} m \frac{1}{\sqrt{1-x^2}}$	1	
		$\sqrt{1-x^2}\frac{dy}{dx} = my -------(1)$	½	
		$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1}{2\sqrt{1-x^2}}(0-2x) = m\frac{dy}{dx}$	1	
		$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m\sqrt{1-x^2}\frac{dy}{dx}$	½	
		$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m.my ----- by(1)$	½	
		$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - m^2y = 0$	½	04
		OR		
		$y = e^{m\sin^{-1}x}$		
		$\frac{dy}{dx} = e^{m\sin^{-1}x} m \frac{1}{\sqrt{1-x^2}}$	1	
		$\sqrt{1-x^2}\frac{dy}{dx} = me^{m\sin^{-1}x}$		
		$\sqrt{1-x^2}\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{1}{2\sqrt{1-x^2}}(0-2x) = me^{m\sin^{-1}x} m \frac{1}{\sqrt{1-x^2}}$	1½	
		$\left(1-x^2\right)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = m^2y$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$\left(1-x^2\right)\frac{d^2y}{dx^2}-x\frac{dy}{dx}-m^2y=0$	½	04
		--		
	b) Ans	If $x=a(\theta+\sin\theta),\ y=a\left(1+\cos\theta\right)$ find $\frac{d^2y}{dx^2}$ at $\theta=\frac{\pi}{2}$		
		$x=a(\theta+\sin\theta),\ y=a\left(1+\cos\theta\right)$		
		$\frac{dx}{d\theta}=a\left(1+\cos\theta\right)-------(1)$	½	
		$\frac{dy}{d\theta}=-a\sin\theta$	½	
		$\therefore\ \frac{dy}{dx}=\frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}}$		
		$\frac{dy}{dx}=\frac{-a\sin\theta}{a\left(1+\cos\theta\right)}$	½	
		$\frac{dy}{dx}=\frac{-2\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\theta}{2}\right)}{2\cos^2\left(\frac{\theta}{2}\right)}$		
		$\frac{dy}{dx}=-\tan\left(\frac{\theta}{2}\right)$	1	
		$\therefore\ \frac{d^2y}{dx^2}=-\sec^2\left(\frac{\theta}{2}\right)\frac{1}{2}\frac{d\theta}{dx}$	½	
		$\frac{d^2y}{dx^2}=-\sec^2\left(\frac{\theta}{2}\right)\frac{1}{2}\frac{1}{\frac{dx}{d\theta}}$		
		$\frac{d^2y}{dx^2}=-\sec^2\left(\frac{\theta}{2}\right)\frac{1}{2}\frac{1}{a\left(1+\cos\theta\right)}$		
		$\frac{d^2y}{dx^2}=-\sec^2\left(\frac{\theta}{2}\right)\frac{1}{2}\frac{1}{a2\cos^2\left(\frac{\theta}{2}\right)}=-\frac{1}{4a}\sec^4\left(\frac{\theta}{2}\right)$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		at $\theta = \frac{\pi}{2}$		
		$\frac{d^2y}{dx^2} = -\frac{1}{4a}\sec^4\left(\frac{\pi}{4}\right) = -\frac{1}{4a}\left(\sqrt{2}\right)^4$	½	
		$\frac{d^2y}{dx^2} = \frac{-1}{a}$	½	04
		--		
	c)	Using Regula-Falsi method, find the root of the equation		
		$x^3 - x - 1 = 0$		
	Ans.	Let $\therefore x^3 - x - 1 = 0 = 0$ $f(x) = x^3 - x - 1$		
		$f(1) = -1 < 0$ $f(2) = 5 > 0$	1	
		$\therefore$ the root lies in (1,2)		
		$x_1 = \frac{af(b) - bf(a)}{f(b) - f(a)} = \frac{1(5) - 2(-1)}{5+1} = 1.167$	1	
		$f(x_1) = -0.578$ <0		
		$\therefore$ the root lies in (1.167,2)		
		$x_2 = \frac{1.167(5) - 2(-0.578)}{5 + 0.578} = 1.253$	1	
		$f(x_2) = -0.286 < 0$		
		$\therefore$ the root lies in (1.253,2)		
		$x_3 = \frac{1.253(5) - 2(-0.286)}{5 + 0.286} = 1.293$	1	04
		OR		
		$\therefore x^3 - x - 1 = 0$ $f(x) = x^3 - x - 1$		

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$f(1) = -1 < 0$ $f(2) = 5 > 0$ $\therefore$ the root lies in (1,2)	1	

a	b	$f(a)$	$f(b)$	$x = \dfrac{af(b) - bf(a)}{f(b) - f(a)}$	$f(x)$
1	2	-1	5	1.167	-0.578
1.167	2	-0.578	5	1.253	-0.286
1.253	2	-0.286	5	1.293	---

Marks: 1+1+1 — Total Marks: 04

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d) Ans

Solve the following equations by Jacobi's method.

$20x + y - 2z = 17, 3x + 20y - z = -18, 2x - 3y + 20z = 25$

$x = \frac{1}{20}(17 - y + 2z)$

$y = \frac{1}{20}(-18 - 3x + z)$

$z = \frac{1}{20}(25 - 2x + 3y)$ — 1

Starting with $x_0 = y_0 = z_0 = 0$

$x_1 = 0.85$
$y_1 = -0.9$
$z_1 = 1.25$ — 1

$x_2 = 1.02$
$y_2 = -0.965$
$z_2 = 1.03$ — 1

$x_3 = 1.001$
$y_3 = -1.002$
$z_3 = 1.003$ — 1

Total Marks: 04

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)	e) Ans	Solve the following equations by Gauss elimination method: $4x+y+2z=12, -x+11y+4z=33, 2x-3y+8z=20$ $4x+y+2z=12$ $-x+11y+4z=33$ $2x-3y+8z=20$		
		$4x+y+2z=12$ $-4x+44y+16z=132$ $+$ ________________ $45y+18z=144$ and $4x+y+2z=12$ $4x-6y+16z=40$ $-\quad+\quad-\quad-$ $7y-14z=-28$ $\therefore\ y-2z=-4$	1	
		$45y+18z=144$ $9y-18z=-36$ ____________ $54y=108$ $\therefore\ y=2$ $z=3$ $x=1$	 1 1 1	04
		Note: *In the above solution, first x is eliminated and then z is eliminated to find the value of y first. If in case the problem is solved by elimination of another unknown i. e., either first y or z, appropriate marks to be given as per above scheme of marking* ---		
	f) Ans	Using Newton-Raphson method to evaluate $\sqrt[3]{20}$ correct to three decimal places. $x=\sqrt[3]{20}$ Let, $\therefore\ x^3-20=0$ $f(x)=x^3-20$ $f(2)=-12<0$ $f(3)=7>0$	 ½	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$f'(x) = 3x^2$		
		$\therefore$ Initial root $x_0 = 3$	½	
		$f'(3) = 27$		
		$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 3 - \frac{(3^3 - 20)}{3(3)^2} = 2.740$	1	
		$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 2.74 - \frac{((2.74)^3 - 20)}{3(2.74)^2} = 2.714$	1	
		$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = 2.714 - \frac{((2.714)^3 - 20)}{3(2.714)^2} = 2.714$	1	04
		OR		
		Let, $x = \sqrt[3]{20}$		
		$\therefore x^3 - 20 = 0$		
		$f(x) = x^3 - 20$	½	
		$f(2) = -12 < 0$		
		$f(3) = 7 > 0$	½	
		$f'(x) = 3x^2$		
		$\therefore$ Initial root $x_0 = 3$		
		$x_{n+1} = \frac{xf'(x) - f(x)}{f'(x)}$		
		$x_{n+1} = \frac{x(3x^2) - (x^3 - 20)}{3x^2}$		
		$x_{n+1} = \frac{3x^3 - x^3 + 20}{3x^2 - 4}$		
		$x_{n+1} = \frac{2x^3 + 20}{3x^2}$	1	

Que. No.	Sub. Que.	Model Answers	Marks	Total Marks
6)		$n = 0,1,2$ $x_1 = 2.740$ $x_2 = 2.714$ $x_3 = 2.714$	 1 ½ ½	 04
		--- ***Important Note*** *In the solution of the question paper, wherever possible all the possible alternative methods of solution are given for the sake of convenience. Still student may follow a method other than the given herein. In such case, first see whether the method falls within the scope of the curriculum, and then only give appropriate marks in accordance with the scheme of marking.* --- ---		